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RESPONSE OF A TURBULENT BOUNDARY LAYER TO A SUDDEN DECREASE IN WALL HEAT FLUX

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Abstract—Measurements are presented of mean and fluctuating temperature fields downstream of a sudden decrease in wall heat flux in a zero pressure gradient turbulent boundary layer. The growth rate of the thickness of the internal layer, estimated from r.m.s. temperature profiles, is found to be in good agreement with that obtained for a sudden increase in wall heat flux. When the internal thermal layer thickness and maximum temperature difference across the layer are used as the normalising length scale and temperature scale respectively, mean, r.m.s. and higher order moments of the temperature are approximately self-preserving, at least over the outer region of the internal layer. The sharp temperature jump at the back of the large structure is observed over a significant part of the boundary layer both upstream and downstream of the sudden change in heat flux.

NOMENCLATURE

Q_w,	thermometric wall heat flux [°C m/s];
T,	mean temperature [°C];
T_1,	free stream temperature [°C];
T_m,	local maximum temperature [°C];
T_w,	wall temperature [°C];
T_τ,	Q_w/U_τ, friction temperature [°C];
t,	time [s];
U,	local mean velocity [m/s];
U_1,	free stream velocity [m/s];
U_τ,	friction velocity [m/s];
u,	streamwise velocity fluctuation [m/s];
v,	normal velocity fluctuation [m/s];
$\overline{v\theta}$,	thermometric heat flux [°C m/s];
x,	streamwise distance [m];
x_s,	streamwise distance, measured from the step [m];
y,	distance normal to the wall [cm];
δ,	kinematic boundary layer thickness [cm];
δ_0,	boundary layer thickness at the step [cm];
δ_i,	internal layer thickness downstream of the step [cm];
θ,	temperature fluctuation [°C];
θ',	$\overline{\theta^2}^{1/2}$, r.m.s. temperature [°C];
κ,	Kármán constant (= 0.41);
τ,	$-\overline{uv}$, kinematic Reynolds shear stress [m^2/s^2];
ν,	kinematic viscosity [m^2/s].

INTRODUCTION

THERE have been several experimental and theoretical investigations of the turbulent boundary layer downstream of sudden changes in surface roughness, primarily because of their relevance to the atmospheric boundary layer. Townsend [1] considered the case of step changes in surface roughness or heat flux, these occurring either separately or in combination. Although the sudden change in surface roughness has been studied experimentally in the laboratory by several investigators (see Antonia and Luxton [2, 3] for references), there have been fewer laboratory investigations of a sudden change in surface heat flux. Johnson [4], Blom [5] and Antonia, Danh and Prabhu [6] considered the response of a smooth wall turbulent boundary layer which is subjected to a sudden increase in surface heat flux or surface temperature. This latter configuration is of some relevance to the often encountered atmospheric situation of a sea breeze which blows onto land. In this situation, the change in surface temperature is inevitably accompanied by a change in surface roughness and the combination tends to create a relatively complicated stratification of the air flow (e.g. Ogawa, Griffiths and Hoydysh [7]). Field studies of this situation have been made by Vugts and Businger [8] and Tieleman and Derrington [9]. Charnay *et al.* [10] measured both mean and fluctuating temperature fields downstream of a sudden decrease in wall temperature. In the present investigation, the wall heat flux Q_w is kept constant upstream of the change while zero heat flux is maintained downstream. This investigation is only of nominal relevance to the atmospheric situation since the surface is smooth everywhere and buoyancy effects are not considered. It should however provide a useful input for methods of calculating the turbulent boundary layer downstream of such a change in surface condition. In particular, the growth of the internal layer and the mean and r.m.s. temperature distributions within the internal layer are discussed in the context of a self-preserving development of the thermal disturbance resulting from the change in boundary condition.

EXPERIMENTAL SET-UP AND CONDITIONS

The boundary layer was developed, under zero pressure gradient, over the smooth wall of the working section (0.38 × 0.23 m) of the wind tunnel. The first 3 m section of the working section floor was heated while the last 1.83 m section was unheated and constructed from an insulating material (0.025 m thick 'Sindanyo' hard asbestos board with an epoxy coating) polished to a smooth flat surface. Immediately upstream of the sudden change in surface condition $T_w - T_1 \simeq 10$°C and T_r is 0.34°C. At this location, the momentum and thermal layers had approximately the same thickness $\delta_0 \simeq 6.3$ cm. Reynolds shear stress and heat flux profiles measured both at this location and upstream of this location indicated that the boundary layer was self-preserving upstream of the change in surface condition. The integral $\int_0^\delta U(T - T_1)\,\mathrm{d}y$ remained approximately constant downstream of the step, which is consistent with the boundary condition $Q_w = 0$.

Mean and r.m.s. temperature measurements were made with a 0.6 μm Pt-10% Rh cold wire (temperature coefficient 1.5×10^{-3} °C^{-1}) operated by a constant current anemometer. The magnitude of the current was sufficiently small (50 μA) for the velocity sensitivity of the cold wire to be negligibly small. Mean and r.m.s. voltages were measured with a DISA 55D31 digital voltmeter and a DISA 55D35 r.m.s. meter respectively. Higher order moments of temperature fluctuations were obtained by digitising the signal recorded on a Hewlett–Packard FM3960A recorder at a sampling frequency of 4 kHz and then processing it on a PDP 11/20 computer. Some insight into the coherent structure of the boundary layer both upstream and downstream of the step has been obtained with the use of a rake of 11 cold wire probes. In the construction of the rake, use was made of a double-sided printed circuit board which supports the probes. Each probe is made by soldering 0.5 mm dia. brass pins which form the prongs, to either side of a narrow (2 × 1 mm) strip of printed circuit board. Experimental results were obtained at streamwise stations x_s/δ_0 of 0.19, 0.54, 1.69, 2.94, 5.73, 8.75, 12.83 and 20.17 at $U_1 = 14.5\,\mathrm{ms}^{-1}$. A schematic representation of the experimental configuration is shown in Fig. 1.

RESULTS AND DISCUSSION

Since Q_w is zero downstream of the step, the location of the maximum in the mean temperature profile occurs at increasing values of y as x_s increases. Antonia and Luxton [2, 3] determined δ_i for a boundary layer subjected to a step change in surface roughness, by inferring the 'merge' point between consecutive mean velocity profiles. A similar approach was used here to find δ_i for the internal layer. Due to experimental scatter in mean temperature profiles, only a rough estimate of δ_i was possible. A more satisfactory estimate of δ_i was inferred from merge points of r.m.s. temperature profiles with the upstream undisturbed profile, as shown in Fig. 2. The abscissa is normalised with the local boundary layer thickness, to allow for the small streamline displacement effect that accompanies the growth of the boundary later. δ_i is taken as the distance from the wall at which the slope of the θ' profile downstream of change matches that of the θ' profile at $x_s/\delta_0 \simeq -0.63$.

The experimental data for δ_i are shown in Fig. 3, with the fit $\delta_i \simeq 0.61\,x_s^{0.64}$. There is only moderate agreement between the present slope and that inferred from Charnay *et al.*'s [10] four values of δ_i. Interestingly, Antonia *et al.* [6] obtained $\delta_i \sim x_s^{0.64}$, where δ_i was taken to be the distance from the wall at which $\theta' = 0.01\,(T_w - T_1)$, for the growth of the internal layer downstream of a sudden increase in wall heat flux. The rate of propagation of the thermal disturbance seems to be independent of the nature of the change in boundary condition. This is not too surprising since the momentum boundary layer is unaffected by the change in surface condition and the initial state of the boundary layer is the same for both the increase and decrease in Q_w (note that the magnitude of the difference between local and free stream temperatures is such that the introduction of temperature does not affect the dynamics of the flow). In contrast, it should be remembered that observations downstream of a step change in roughness (Antonia and Luxton [2, 3]) indicated that $\delta_i \sim x_s^{0.79}$ for a smooth to rough step and $\delta_i \sim x_s^{0.43}$ for a rough to smooth step.

To estimate δ_i, Townsend's [1] conditions for self-preservation of the mean temperature cannot be used effectively in the present case as thermal equilibrium is not likely to be satisfied in the region close to the wall (the gradient $\partial T/\partial y$ is zero at the wall). The calculation method of Bradshaw and Ferriss [11] yields an equation for the outgoing characteristic from $x_s = 0$,

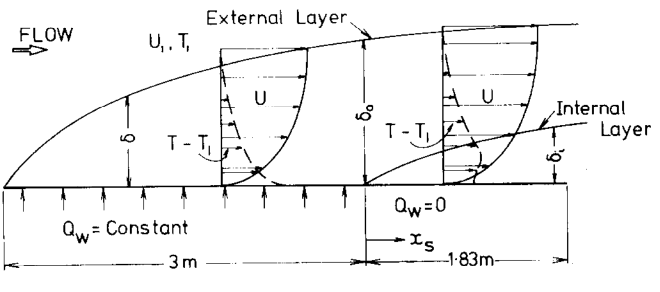

FIG. 1. Sketch of experimental arrangement and conditions.

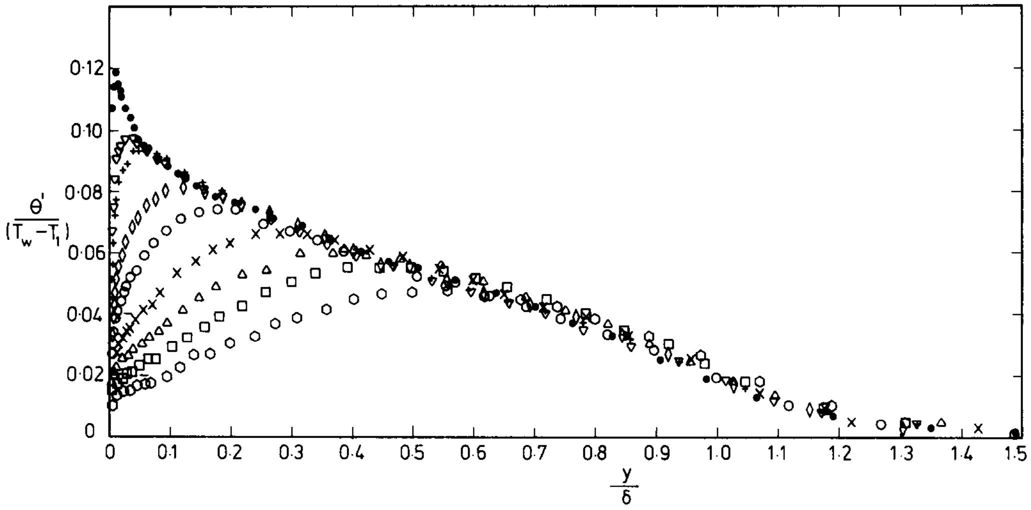

FIG. 2. R.m.s. temperature profiles. ●, $x_s/\delta_0 = -0.63$; ▽, 0.19; +, 0.54; ◇, 1.69; ○, 2,94; ×, 5.73; △, 8.75; □, 12.83; ◊, 20.17.

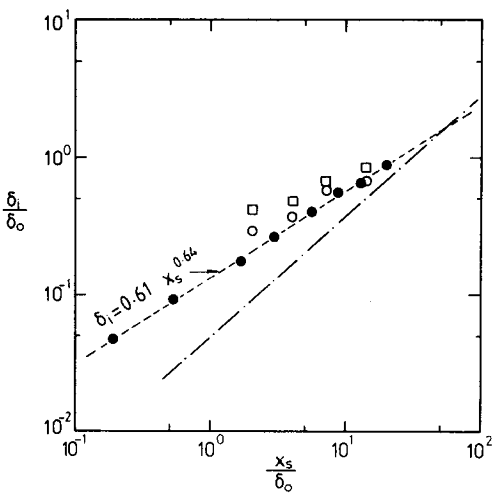

FIG. 3. Growth of the internal layer δ_i. ●, present data. Values of Charnay *et al.*'s [10] δ_i values: ○, from mean temperature profile; □, from r.m.s. temperature profile. — · — · —, Bradshaw and Ferris [11] calculation.

provided advection and diffusion of $\overline{\theta'^2}$ are negligible. These terms are small compared with the production and dissipation terms in the outer region of the internal layer in the $\overline{\theta'^2}$ budget of Charnay *et al.* [10]. Identifying the lateral position of the characteristic with δ_i this equation can be written as

$$\frac{d\delta_i}{dx_s} = \frac{(a_{1\theta}^2 \tau)^{1/2}}{U}, \qquad (1)$$

where $a_{1\theta}$ is a structure function parameter defined by

$$a_{1\theta} = \frac{\overline{v\theta}}{\theta' \tau^{1/2}}. \qquad (2)$$

The logarithmic distribution of U is given by

$$\frac{U}{U_\tau} = \kappa^{-1} \ln \frac{y U_\tau}{\nu} + C, \qquad (3a)$$

where the von Kármán constant $\kappa \simeq 0.41$ and $C \simeq 5.6$. Alternatively (3a) can be re-written as

$$\frac{U}{U_\tau} = \kappa^{-1} \ln \frac{y}{z_0}, \qquad (3b)$$

where the roughness length $z_0 = \nu\, e^{-\kappa C}/U_\tau$. Using (3b), equation (1) can be integrated in the streamwise direction starting at $x_s = 0$ when $\delta_i \simeq z_0$. This integration yields

$$\delta_i[\ln(\delta_i/z_0) - 1] + z_0 = \kappa\, a_{1\theta}\, x_s$$

with the assumption that $\tau^{1/2} \simeq U_\tau$. At values of x_s large enough to satisfy $\delta_i \gg z_0$ but small enough for δ_i to lie within the logarithmic region, the previous equation can be approximated by

$$\delta_i[\ln(\delta_i/z_0) - 1] \simeq \kappa\, a_{1\theta}\, x_s. \qquad (4)$$

Distributions of $a_{1\theta}$ (Fig. 4) have been inferred for Charnay *et al.*'s data, using Klebanoff's [12] distribution for τ. Although $a_{1\theta}$ is not constant, it is independent of x_s in the internal layer when plotted against y/δ_i. Antonia *et al.* [6] also found that $a_{1\theta}$ does not change appreciably with x_s, but, in their case, $a_{1\theta}$ was constant, approx. 0.64, over a significant region of the internal layer. The calculated distribution of δ_i shown in Fig. 3, is obtained from equation (4) using $a_{1\theta} = 0.7$, the value estimated at $y = \delta_i$. The calculated growth rate of δ_i is appreciably larger than the experimental growth rate. It should however be noted that the requirement, implicit in equation (4), that δ_i is small compared with the thickness of the logarithmic region is contravened at relatively small values of

x_s/δ_0. Figure 3 indicates that δ_i exceeds 0.2 δ_0 when x_s is greater than about 2 δ_0.

Mean temperature profiles downstream of the step in Fig. 5 are approximately self-preserving in the region $0.05 < y/\delta_i < 0.6$, when δ_i is used as the relevant length scale and $(T - T_1)$ is normalised by $(T_m - T_1)$. The temperature profile has zero slope at $y/\delta_i \simeq 0.1$, for $y/\delta_i < 0.05$, profiles cannot be expected to be self-preserving because the production of θ'^2 is small, and the flow is not in local equilibrium, as evidenced by Charnay *et al.*'s budget of θ'^2. Charnay *et al.*'s mean temperature profile at $x_s/\delta_0 = 7.14$ shows a peak at $y/\delta_i \simeq 0.25$ and the profile is rather steeper close to the wall.

When θ' is normalised by $(T_w - T_1)$, the resulting distributions (not shown here) downstream of the step exhibit approximate self-preservation for $0.05 < y/\delta_i < 1.4$. The maximum value occurs at $y/\delta_i \approx 0.8$, which corresponds roughly to the location where the slope of the mean temperature profile is maximum. Charnay *et al.* observed a second maximum in the production term of θ'^2 close to the wall, outside the viscous sublayer.

Since the signal to noise ratio of θ decreases with x_s, the internal layer interface could not be studied in detail and conditional measurements associated with this interface were not attempted. However, a few normalised high order moments $\overline{\theta^n}/\theta'^n$ $(n = 3\text{–}6)$ were evaluated at $x_s/\delta_0 = 8.75$ and 20.17 (Fig. 6). The distributions of these moments are in good agreement at the two values of x_s for $y < \delta_i$. This supports previous conclusions, drawn from the mean and r.m.s. temperature profiles, in relation to the self-preservation of the internal layer. The skewness (n = 3) is positive in the region $y/\delta_i < 0.17$ presumably because of the arrival in this region of relatively warmer fluid from the outer part of the internal layer. The negative skewness in the region $0.017 < y/\delta_i < 0.8$

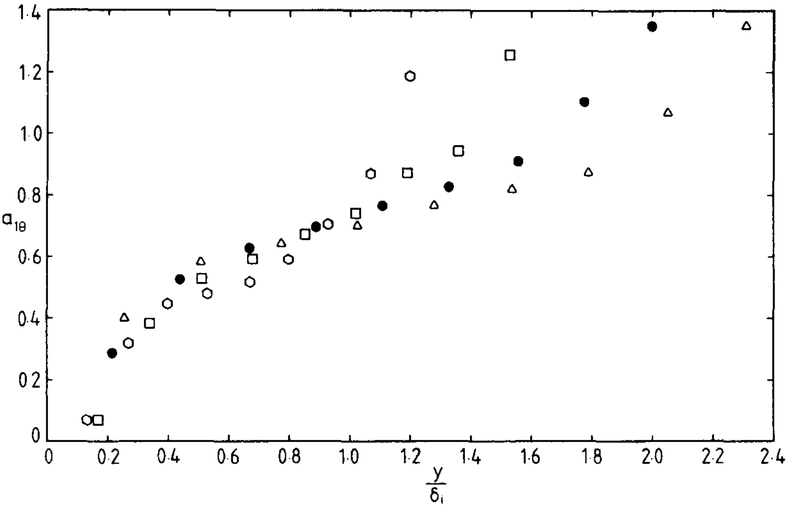

FIG. 4. Distributions of $a_{1\theta}$ using data of Charnay *et al.* [10]. Δ, x_s/δ_0 = 2.04; ●, 4.08; □, 7.14; ○, 14.29.

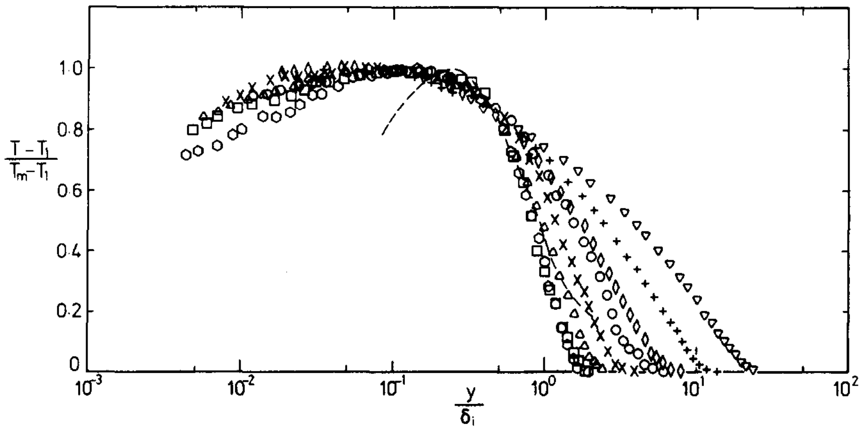

FIG. 5. Normalised mean temperature profile. Symbols same as in Fig. 2. ---, Charnay *et al.* [10] $x_s/\delta_0 = 7.14$.

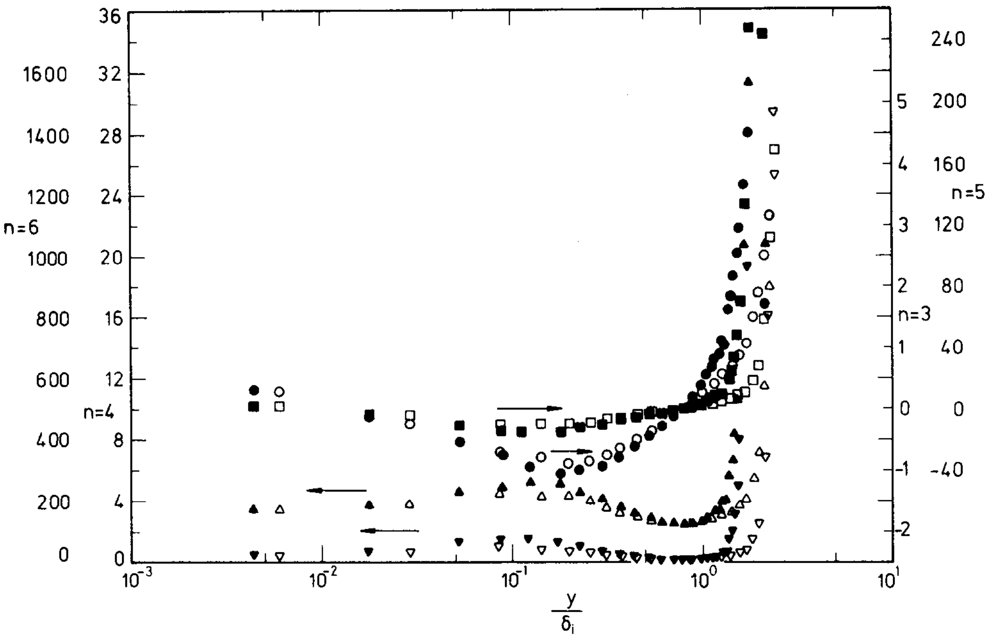

FIG. 6. Distributions of $\overline{\theta^n}/\theta'^n$. ○, $n = 3$; △, 4; □, 5; ▽, 6. Open symbols $x_s/\delta_0 = 8.75$; closed symbols $x_s/\delta_0 = 20.17$.

may reflect the arrival of relatively cooler fluid from the wall region or from the outer part of the boundary layer. The distribution of $\overline{\theta^5}/\theta'^5$ is in qualitative agreement with that of $\overline{\theta^3}/\theta'^3$. In the external part of the boundary layer, both even and odd order moments increase rapidly as a result of outer layer intermittency. The fourth and sixth order moments exhibit a minimum near $y = \delta_i$. At this location, the even order moments are approximately zero. It is of interest to note that Antonia and Luxton [13] observed maxima for the skewness and flatness factors of u at $y \approx \delta_i$ for a smooth to rough step change. Maxima were not found for a rough to smooth step near $y = \delta_i$, presumably because the statistics of u near the edge of the internal layer were dominated by the relatively high turbulence intensity of the upstream rough wall boundary layer.

Temperature traces (Figs. 7b, c) obtained with the rake downstream of the step show the same jumps at the back of the large structure that have been observed (e.g. Fig. 7a) in a continuously slightly heated flow (also Chen and Blackwelder [14]; Subramanian and Antonia [15]). Although the temperature at the wall is approximately equal to that in the free stream, the temperature jump is from hot to cold at the back of the large structure, in the layer near the wall as well as in the outer layer. Chen and Blackwelder [14] described the region associated with a sudden decrease in temperature as separating relatively high speed fluid upstream of this region from relatively low speed fluid downstream. These authors identified this thermal interface with an 'internal shear layer', characterised by relatively sharp changes in both streamwise and normal velocities. The thermal interface provides a possible mechanism by which the large structure in the outer intermittent region is related to the bursting phenomenon near the wall. Charnay *et al.* [10, Fig. 22] showed simultaneous traces, at a particular value of x_s, of u (in the outer part of the layer) and θ near the edge of the internal layer downstream of an increase in surface temperature. Both the turbulent/irrotational interface and internal layer interface register the passage of the back of the structure. The relatively sudden increase in u precedes the sharp decrease in θ at the edge of the internal layer. This is quite consistent with the inclination due to the shear of the large structure. In Fig. 7c, the sharp decrease in θ certainly penetrates the internal layer, as the mean position of this layer is located at $\delta_i \simeq 0.67\,\delta$. The inclination of the thermal interface to the wall is clearly evident in all sets of traces shown in Fig. 7.

CONCLUSIONS

The main conclusions of the present investigation can be summarised as follows:

(i) The thickness δ_i of the internal layer that develops downstream of a sudden decrease in wall heat flux increases as $x_s^{0.64}$, where x_s is measured from the change. A similar growth rate, also based on estimates of δ_i inferred from r.m.s. temperature profiles, was observed for the internal layer downstream of a sudden increase in wall heat flux.

(ii) Mean and r.m.s. temperature profiles are approximately self-preserving over a significant part of

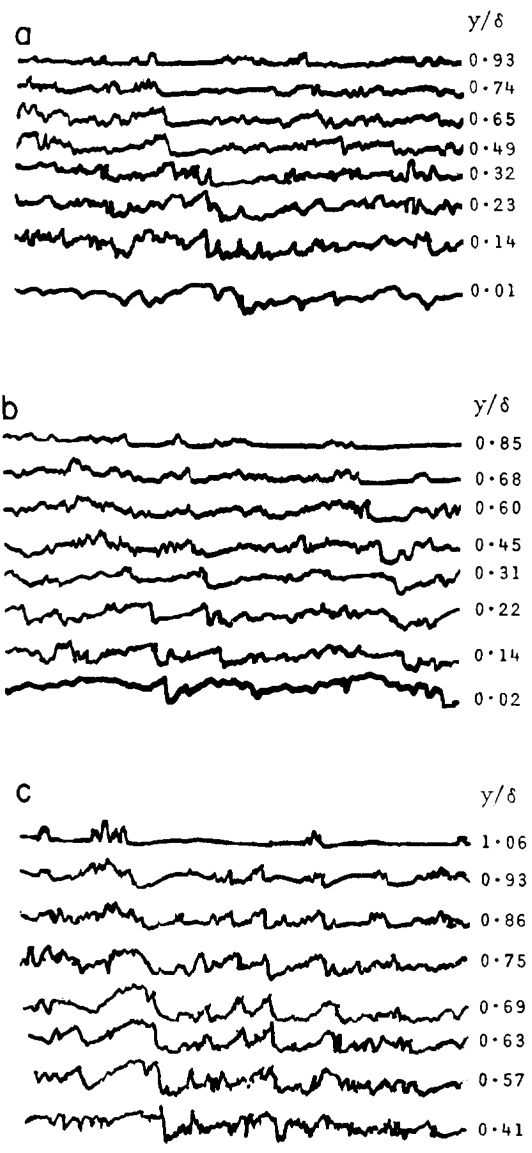

FIG. 7. Typical temperature traces obtained with the cold wire rake. (a) $x_s/\delta_0 = -4.33$, the lowest trace corresponds to $yU_\tau/\nu \approx 21$. Flow is right to left. (b) $x_s/\delta_0 = 1.6$, the lowest trace corresponds to $yU_\tau/\nu \approx 55$; $\delta_i/\delta \approx 0.17$. (c) $x_s/\delta_0 \approx 20.17$; the lowest trace corresponds to $\delta_i/\delta \approx 0.67$.

the internal layer when δ_i and the maximum mean temperature difference are used as the appropriate normalising length and temperature scales respectively. Even order moments of the temperature fluctuations are approximately zero at $y = \delta_i$.

(iii) The dynamics of the flow downstream of the change in surface condition continue to be dominated by the organised large structure of the motion. The upstream interface of this structure, characterised by a sudden decrease in temperature, is first observed in the outer part of the layer but extends across the internal layer into the wall region of the flow.

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REPONSE D'UNE COUCHE LIMITE TURBULENTE A UNE DECROISSANCE SOUDAINE DANS UN FLUX PARIETAL

Résumé—On présente des mesures de températures moyenne et fluctuante en aval d'une décroissance brutale de flux pariétal dans une couche limite turbulente sans gradient de pression. La vitesse de croissance de l'épaisseur de la couche interne, estimée à partir du profil de moyenne quadratique de température, est trouvée en bon accord avec celle obtenue pour une croissance soudaine du flux pariétal. Quand on utilise l'épaisseur de la couche thermique interne et la différence maximale de température à travers la couche respectivement comme échelles de longueur et de température, la moyenne, la moyenne quadratique et les moments plus élevés de la température sont approximativement établis, au moins dans la région externe de la couche interne. Le saut de température à l'arrière de la grande structure est observé sur une partie significative de la couche limite, à la fois en amont et en aval du changement brutal du flux thermique.

VERHALTEN EINER TURBULENTEN GRENZSCHICHT BEI PLÖTZLICHER ABNAHME DER WÄRMESTROMDICHTE IN DER WAND

Zusammenfassung—Es werden Messungen von zeitlich gemittelten und fluktuierenden Temperaturfeldern stromabwärts von einer Stelle plötzlicher Abnahme der Wand-Wärmestromdichte in einer turbulenten Grenzschicht ohne Druckgradienten mitgeteilt. Die Dickenzunahme der inneren Schicht, abgeschätzt aus nach der Fehlerquadratmethode gemittelten Temperaturprofilen, stimmt gut mit jener überein, die man für eine plötzliche Zunahme der Wärmestromdichte an der Wand erhält. Wenn man die Dicke der inneren thermischen Schicht und die maximale Temperaturdifferenz aus der Schicht als normierende Längenskala bzw. Temperaturskala verwendet, sind der quadratische Mittelwert und die Momente höherer Ordnung der Temperatur nahezu konstant, zumindest über die äußere Region der inneren Schicht. Der steile Temperatursprung an der Rückseite der großen Struktur wird über einen signifikanten Teil der Grenzschicht sowohl stromauf- als auch stromabwärts von der Stelle des plötzlichen Wechsels der Wärmestromdichte beobachtet.

ВЛИЯНИЕ МГНОВЕННОГО СНИЖЕНИЯ НАГРЕВА СТЕНКИ НА ПОВЕДЕНИЕ ТУРБУЛЕНТНОГО ПОГРАНИЧНОГО СЛОЯ

Аннотация — Представлены результаты измерений средних и пульсационных значений температуры вниз по течению за областью мгновенного снижения величины подводимого к стенке теплового потока в турбулентном пограничном слое при нулевом градиенте давления. Найдено, что скорость роста толщины пристенного слоя, рассчитанная по среднеквадратичным профилям температур, хорошо согласуется со значениями, полученными при мгновенном увеличении нагрева стенки. В случае, когда толщина пристенного теплового слоя и максимальная разность температур поперек слоя используются соответственно как нормировочные масштабы длины и температуры, средние и среднеквадратичные значения температур, а также моменты более высоких порядков почти не изменяются, по крайней мере, во внешней области пристенного слоя. Вблизи стенки, на большом протяжении пограничного слоя как вверх, так и вниз по течению от области внезапного изменения величины подводимого теплового потока, наблюдается острый температурный скачок.